Midterm Exam Calculus 1

24 september 2012, 9.00-11.00.

Write on each page your name and student number, and on the first page your seminar group. The use of annotations, books and calculators is not permitted in this examination. All answers must be supported by arguments/work. Success.

1. (a) Formulate the principle of mathematical induction.

   (b) Prove that if $n \geq 1$ is a positive integer, then $13^n - 6^n$ is divisible by 7.

2. Find all (complex) solutions of
$$z^2 = \frac{1}{1 - i}$$
   and plot them in the complex plane.

3. Determine all complex numbers z satisfying
$$e^z = -1$$

4. The function $f(x)$ is defined fo$-\infty < x < \infty$.

   (a) Give the ϵ-δ-definition of
$$\lim_{x \to \infty} f(x) = L$$

   (b) Prove (using this definition) that
$$\lim_{x \to \infty} \frac{1}{x} = 0$$

Maximum score:

1a	1.0	2	2.0	3	2.0	4a	1.0
b	2.0					b	1.0

Total: 9 +1 (free) = 10.